PROBLEMS.

44 Proposed by I. J. SCHWATT, Ph. D.. Professor of Mathematics, University of Pennsylvania, Philadelphia, Pennsylvania.

(1). If from the middle point M of the side BC of the triangle ABC a parallel to the bisector AF of the external angle to ABC is drawn to meet AB at K, the point K divides then the side AB in KA

$$=\tfrac{1}{2}(AB+AC) \text{ and } KB=\tfrac{1}{2}(AB-AC).$$

(2). If K is joined to the extremity D of the diameter perpendicular to BC then is KD perpendicular to AB.

45. Proposed by B. F. BURLESON, Oneida Castle, New York.

Determine the radius of a circle circumscribing three tangent circles of radii $a=15$, $b=17$, and $c=19$.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

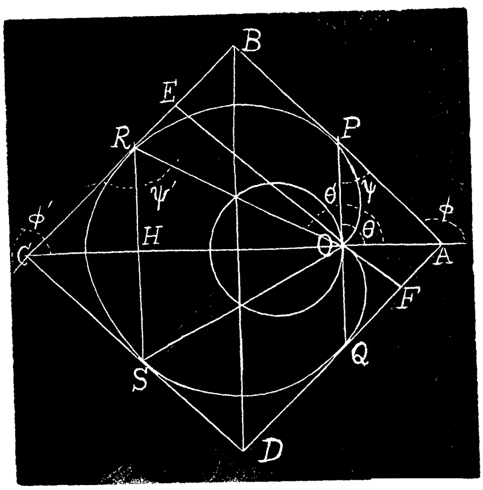
SOLUTIONS OF PROBLEMS.

30. Proposed by E. W. NICHOLS, Professor of Mathematics in the Virginia Military Institute, Lexington, Virginia.

Given the cardioid $r=a\,(1-\cos\theta)$; find the area of its circumscribing square formed by tangents making angles of 45° with its axis.

I. Solution by Professor G. B. M. ZERR. A. M., Principal of High School, Staunton, Virginia.

Let $OPRSQ$ be the cardioid. Draw PQ through the cusp perpendicular to the initial line AC. From the property of the cardioid the angle APO, made by the tangent and radius vector at P, $=\frac{1}{2}\angle POA$. But $\angle AOP=\frac{1}{2}\pi$. $\therefore$ $\angle OPA=\angle OAP$ $=\frac{1}{4}\pi$. $\therefore$ the tangents BA, DA at the points P, Q are inclined at an angle of 45° to the axis and are perpendicular to each other. Draw the radii vectors OR, OS, making the $\angle ROP=\angle SOQ=\frac{1}{3}\pi$, and draw the tangents CB, CD at the points R, S. Then $\angle ROP=\angle SOQ=\frac{1}{3}\pi$, $\angle OPB$ $=\angle OQD=\frac{3}{4}\pi$, $\angle ORB=\angle OSD$ $=\frac{5}{12}\pi$.



$\therefore$ $\angle ROP+\angle OPB+\angle ORB$ $=\angle SOQ+\angle OQD+\angle OSD=\frac{3}{2}\pi$.

$\therefore$ $\angle B=\angle D=\frac{1}{2}\pi$, an $ABCD$ is the required square.